

**Solution to Problem 10)** a) With reference to Problem 5, the second derivative of  $f(x) = e^x$  is  $f''(x) = e^x$ , which is positive everywhere. Therefore,  $e^x$  is a convex cup function. The second derivative of  $g(x) = \ln x$  is  $g''(x) = -1/x^2$ , which is negative on the positive  $x$ -axis. Therefore,  $\ln x$  is a convex cap function.

b) Invoking Jensen's inequality, we conclude that, for the convex cup function  $e^x$ , one must have  $\langle e^x \rangle \geq e^{\langle x \rangle}$ , and for the convex cap function  $\ln x$ , one must have  $\langle \ln x \rangle \leq \ln \langle x \rangle$ .

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